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TECHNICAL INFORMATION SECTION

The theory of thermal explosions: the initiation of  
explosion by a solid spherical hot spot

G. B. Cook

F. E. Mauger

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Fort Halstead  
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September  
1958

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17 October 1958

SUBJECT: The Theory of Thermal Explosions; the Initiation of  
Explosion by a Solid Spherical Hot Spot (U)

TO: Commanding Officer  
US Army Ordnance Technical Intelligence Agency  
Arlington Hall Station  
Arlington 12, Virginia

Forwarded herewith as Inclosure #1 is Armament Research & Development Establishment Report (B) 24/58 entitled "The Theory of Thermal Explosions; the Initiation of Explosion by a Solid Spherical Hot Spot", by G.B. Cook and P.E. Meager, dated September 1958.

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ARMAMENT RESEARCH AND DEVELOPMENT ESTABLISHMENT

A.R.D.E. REPORT (B)24/58

The theory of thermal explosions: the initiation of  
explosion by a solid spherical hot spot

G. B. Cook (B1)

F. E. Mauger (B1)

Summary

A small hot sphere of chemically inert material is embedded in a mass of cold explosive. The time to ignition of the explosive is determined by numerical methods, and the way in which this time varies with the properties of the hot spot and explosive is investigated. Four specific hot spot materials are considered: copper, steel, glass and a further material with thermal properties similar to those of the explosive. The first three of these were surrounded by RDX, but, if certain conditions are satisfied, the results are applicable to other explosives.

Approved for issue:

D. H. Black C.M.G., Director

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## 1. INTRODUCTION

In this report, the initiation of an explosion by a small hot sphere of chemically inert material embedded in a mass of cold explosive is considered. The object of the present calculations is twofold: first, to set up and programme the appropriate equations in a finite difference form suitable for numerical integration on the high speed computer AMOS; and second, to evaluate the time to ignition for a given hot spot and explosive and to investigate the effect of the thermal properties of the hot spot material on the ignition time. In order to cover a wide range of explosives an enormous amount of computation would be required; this did not seem justified at present in view of several approximations and assumptions involved in the mathematical model. Only one explosive, RDX, has therefore been treated in detail but it should be pointed out that the programmes constructed for AMOS are quite general and any further cases of particular interest could be easily examined.

A series of experimental investigations of hot spot initiation has been carried out by Bowden and his co-workers and details are given in Bowden and Yoffe [1]. Their general conclusions concerning the initiation of explosion by grit particles were that a particle of about  $10^{-3}$  cms. diameter at about  $500^{\circ}\text{C}$  was capable of causing initiation in a time of the order of  $10^{-4}$  secs. in a small sample of explosive. Roughly speaking, the same orders of magnitude have been found in the present calculations although, as we shall show later, the effect of the thermal properties of the hot spot is quite pronounced. As far as the authors are aware, no previous theoretical attempt to study the effect of a solid inert hot spot has been made. Rideal and Robertson [2] have considered initiation by an explosive hot spot and have evaluated critical radii and temperatures for several explosives; some further work on explosive hot spots was carried out on the Aiken computer at Harvard and is reported by Seager [3]. An approximate discussion of initiation in a plastic explosive due to the rapid compression of enclosed air bubbles has been given by Coombs and Thornhill [4].

The present report is the third of a series in thermal explosion theory. In the first of these [5], one face of a slab of cold explosive was imagined to be placed in contact with a constant temperature bath. The time to ignition was calculated, and from the results, empirical relations connecting the ignition time with the initial uniform temperature of the sample and the bath temperature, were derived. In the second [6], a pulse of high intensity thermal radiation, such as that delivered from an atomic weapon, was assumed to fall on a steel plate covering a mass of explosive; critical conditions under which explosion only just manages to occur were evaluated for RDX. The mathematical details gathered in paras. 3 and 4 may be passed over, if desired, without loss of continuity.

## 2. FORMULATION OF THE PROBLEM

Consider a small hot sphere of radius  $a$  and initial uniform temperature  $T_0$  embedded in an infinite mass of cold explosive at initial uniform temperature  $T_1$ . This model is suitable for quite small samples of explosive since the explosive is a poor conductor and, in the short times involved, the heat penetration is very small. On the assumption that the thermal properties of the hot spot material and the explosive do not vary with temperature and that the decomposition of the explosive can be represented by a single chemical reaction of the zero order, the governing equations can be written,

$$\rho_1 c_1 \frac{\partial T_1}{\partial t} = \frac{k_1}{r^2} (r^2 \frac{\partial T_1}{\partial r}), \quad 0 < r < a \quad (1)$$

$$\rho_2 C_2 \frac{\partial T_2}{\partial t} = \frac{k_2}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T_2}{\partial r} \right) + \rho_2 Q Z e^{-E/RT_2}, \quad r > a \quad (2)$$

where suffixes '1' and '2' refer to the hot spot material and explosive respectively. In (1) and (2) the following notation is employed:

$k$  = thermal conductivity,

$\rho$  = density,

$c$  = specific heat,

$T$  = temperature,

$E$  = activation energy of the reaction,

$Q$  = heat liberated by chemical reaction per gram,

and  $Z$  = frequency factor.

Equations (1) and (2) have to be solved under the following initial and boundary conditions:

$$t = 0; \quad 0 < r < a; \quad T_1 = T_0, \quad (3)$$

$$r > a; \quad T_2 = T_i, \quad (4)$$

$$t > 0; \quad r = a; \quad T_1 = T_2. \quad (5)$$

$$k_1 \frac{\partial T_1}{\partial r} = k_2 \frac{\partial T_2}{\partial r}, \quad (6)$$

$$r \rightarrow \infty; \quad T_2 \rightarrow T_i, \quad (7)$$

$$r = 0; \quad T_1 \text{ finite.} \quad (8)$$

It is convenient to introduce the dimensionless variables  $\xi$ ,  $\tau$  and  $\theta$  defined by

$$\begin{aligned} \theta &= \frac{R}{E} T, \quad \xi = \left( \frac{RQZ\rho_2}{k_2 E} \right)^{1/2} 10^{-7} r, \quad \tau = \left( \frac{RQZ}{C_2 E} \right) 10^{-14} t, \\ &= a_1 T, \quad = a_2 r, \quad = a_3 t, \end{aligned} \quad (9)$$

where we suppose  $\xi$  has the value  $\alpha$  when  $r = a$ .

The resulting equations can be further simplified by writing:

$$u = \xi \theta,$$

and it is also convenient to introduce the variable  $\zeta$ , defined in the interval  $0 < \xi < \alpha$  by the equation

$$\zeta = \sqrt{\frac{K_2}{K_1}} \xi,$$

where  $K$  is the thermal diffusivity  $k/\rho c$ , and  $\zeta = \zeta_0$  when  $\xi = \alpha$ . Equations (1) to (8) then become:

$$0 < \zeta < \zeta_0; \quad \frac{\partial u_1}{\partial \tau} = \frac{\partial^2 u_1}{\partial \zeta^2}, \quad (1')$$

$$\xi > \alpha; \quad \frac{\partial u_2}{\partial \tau} = \frac{\partial^2 u_2}{\partial \xi^2} + 10^{14} \xi e^{-\xi/u_2}, \quad (2')$$

$$\tau = 0; \quad 0 < \zeta < \zeta_0; \quad u_1 = \sqrt{\frac{K_1}{K_2}} \zeta \theta_0, \quad (3')$$

$$\xi > \alpha; \quad u_2 = \xi \theta_1, \quad (4')$$

$$\tau > 0; \quad \xi = \alpha; \quad u_1 = u_2, \quad (5')$$

$$k_1 K_2 \frac{\partial}{\partial \zeta} \left( \frac{u_1}{\zeta} \right) = k_2 K_1 \frac{\partial}{\partial \xi} \left( \frac{u_2}{\xi} \right), \quad (6')$$

$$\xi \rightarrow \infty; \quad u_2 \sim \xi \theta_1, \quad (7')$$

$$\xi = 0; \quad u_1 = 0. \quad (8')$$

It has been stated above that the thermal properties of both materials have been assumed to be independent of temperature. This assumption is mathematically convenient but it should be pointed out that, in general, data on the variation of these properties with temperature is rather scarce. It has been further assumed that, over a wide temperature range, the decomposition can be represented by a single chemical reaction of the zero order. While this is undoubtedly an oversimplification, it is probably sufficiently realistic to enable at least the qualitative features of the behaviour of the system to be predicted. It is further assumed that convection effects arising from the liquefaction of the explosive can be ignored and that, at all times, there is no contact resistance at the explosive/hot spot interface.

### 3. FINITE DIFFERENCE METHOD

Providing a certain restriction is observed, a stable finite difference form of equations (1') and (2') emerges if forward differences are used for the time derivatives, and central differences for the space derivatives. With the definitions:

$$u_{n,j}^{(1)} = u_1 (n\Delta\zeta, j\Delta\tau),$$

$$u_{m,j}^{(2)} = u_2 (\alpha + m\Delta\xi, j\Delta\tau),$$

$$\text{and } \zeta_0 = N\Delta\zeta,$$

equations (1') and (2') become:

$$u_n^{(1)},_{j+1} = \sigma_1 (u_{n+1}^{(1)},_j + u_{n-1}^{(1)},_j - 2u_n^{(1)},_j) + u_n^{(1)},_j, \quad 0 \leq n < N \quad (10)$$

$$u_m^{(2)},_{j+1} = \sigma_2 (u_{m+1}^{(2)},_j + u_{m-1}^{(2)},_j - 2u_m^{(2)},_j) + u_m^{(2)},_j \\ + 10^{14} (\alpha + m\Delta\xi) \Delta\tau \exp \left\{ -(\alpha + m\Delta\xi)/u_m^{(2)},_j \right\}, \quad m > 0 \quad (11)$$

where

$$\sigma_1 = \Delta\tau/(\Delta\xi)^2, \quad \sigma_2 = \Delta\tau/(\Delta\xi)^2.$$

In the numerical work  $\Delta\xi$  and  $\Delta\zeta$  were always given identical values, so that

$$\sigma_1 = \sigma_2 = \sigma.$$

There is still a choice of methods that may be used to expand the boundary condition (6') in finite difference form. Here two methods are developed, and compared empirically. In the first both terms are expanded by Taylor's theorem and central differences used on the resulting expression. This method yields the boundary equation:

$$u_{N,j} = u_0,j = 4A_1 u_{N-1,j} + 4A_2 u_1,j - A_1 u_{N-2,j} - A_2 u_2,j, \quad (12)$$

where

$$A_1 = \mu / \left[ 3\mu + 3\nu - \frac{2(\Delta\xi)}{\alpha} (1 - \nu) \right],$$

$$A_2 = \nu / \left[ 3\mu + 3\nu - \frac{2(\Delta\xi)}{\alpha} (1 - \nu) \right],$$

$$\mu = \sqrt{\frac{K_2}{K_1}},$$

$$\text{and } \nu = \frac{k_2}{k_1}.$$

Alternatively the "hypothetical point" method may be used in which it is assumed that each material is extended over the actual interface by an

amount  $\Delta\xi$ , the value of  $u$  at these points being defined as  $u_{N+1,j}^{(1)*}$  in the

case of the hot spot material and  $u_{-1,j}^{(2)*}$  for the explosive. The equations governing the behaviour of the hypothetical points, corresponding to (1'), (2'), (5') and (6'), are

$$u_{N,j+1}^{(1)} = (u_{N+1,j}^{(1)*} + u_{N-1,j}^{(1)} - 2u_{N,j}^{(1)}) \sigma + u_{N,j}^{(1)},$$

$$u_{0,j+1}^{(2)} = (u_{1,j}^{(2)*} + u_{-1,j}^{(2)*} - 2u_{0,j}^{(2)}) \sigma + u_{0,j}^{(2)} + \alpha \Delta\tau 10^{14} e^{-\alpha/\Delta\xi} u_{0,j}^{(2)},$$

$$(2) \quad (1) \\ u_{0,j} = u_{N,j}$$

and

$$\frac{2}{\alpha} (\Delta\xi)(1 - \nu) u_{N,j}^{(1)} = \mu (u_{N+1,j}^{(1)*} - u_{N-1,j}^{(2)}) - \nu (u_{1,j}^{(2)} - u_{-1,j}^{(2)*}).$$

Eliminating  $u^*$  yields the boundary condition

$$u_{N,j+1}^{(1)} = u_{0,j+1}^{(2)} = 2\sigma \left\{ (1 - B_1) u_{N-1,j}^{(1)} + B_1 u_{1,j}^{(2)} + B_2 u_{N,j}^{(1)} + B_1 \alpha \Delta \tau 10^{14} e^{-\alpha/u_{0,j}^{(2)}} \right\} \quad (12')$$

where

$$B_1 = \frac{\nu}{\mu + \nu},$$

$$B_2 = \frac{1}{2\sigma} - 1 + \frac{\Delta\xi}{\alpha} \frac{(1 - \nu)}{(\mu + \nu)}.$$

The remaining boundary equations in finite difference form, to be used with either (12) or (12'), are

$$u_{m,j}^{(2)} \rightarrow (\alpha + m\Delta\xi) \theta_i, \quad m \rightarrow \infty \quad (13)$$

$$u_{0,j}^{(1)} = 0, \quad (14)$$

$$u_{n,0}^{(1)} = n\Delta\xi \frac{\theta_0}{\mu}, \quad 0 < n < N \quad (15)$$

$$\text{and} \quad u_{m,0}^{(2)} = (\alpha + m\Delta\xi) \theta_i. \quad m > 0 \quad (16)$$

Once an initial line is supplied, equations (10) - (14) are sufficient, for the evaluation of  $u$  at all later times. However, equations (15) and (16) give a discontinuity on the initial line at the interface which is undesirable at the commencement of the computation. To overcome this difficulty a formal solution of the conduction equations in the plane case, and without the exponential term, may be obtained in the form:

$$u_{n,j}^{(1)} = \frac{n(\Delta\xi)}{\mu} \left[ \theta_0 - \frac{\nu}{\nu + \mu} (\theta_0 - \theta_i) \operatorname{erfc} \frac{N - n}{2\sqrt{\nu}} \right], \quad (17)$$

$$u_{m,j} = (\alpha + m\Delta\xi) \left[ \theta_i + \frac{\mu}{\nu + \mu} (\theta_0 - \theta_i) \operatorname{erfc} \frac{m}{2\sqrt{\nu}} \right].$$

In general, one is justified in starting the computation from a small value of  $j$  using equations (17), since initially the temperature in the explosive is sufficiently low for the rate of chemical reaction to be negligible and, for a sufficiently large hot spot, there is little difference between the solutions of the plane and spherical cases.

## 4. MATHEMATICAL DISCUSSION

Typical temperature profiles based on the equations (10)-(17) are shown in Fig. 1. Ignition was said to have occurred, and the time  $t_e$  to ignition derived, when the rate of increase of temperature at any point in the explosive accelerated rapidly.

Before investigating the effect on  $t_e$  of the physical parameters  $a$ ,  $T_0$ ,  $T_i$ ,  $\mu$  and  $\nu$ , it is of interest to determine the way in which these results will depend on the choice of the mathematical parameters  $\Delta\tau$ ,  $\Delta\xi$  and  $\sigma$ , and also on the choice of the boundary equation.

As it is not possible to compare directly the numerical solutions with the formal solution of the general equations, we consider the simpler case of a spherical hot spot embedded in a non-explosive material and with the condition  $\mu = \nu = 1$ . The solution in a suitable form is

$$u_{m,j}^{(1)} = \theta_0 n \Delta\xi - \frac{1}{2} (\theta_0 - \theta_i) \left[ 2 \left( \frac{j \Delta\tau}{\pi} \right)^{1/2} \left\{ e^{-\frac{(N-n)^2}{4\sigma j}} - e^{-\frac{(N+n)^2}{4\sigma j}} \right\} - n \Delta\xi \left\{ \operatorname{erfc} \frac{N-n}{2\sqrt{\sigma j}} - \operatorname{erfc} \frac{N+n}{2\sqrt{\sigma j}} \right\} \right], \quad (18)$$

$$u_{m,j}^{(2)} = \theta_i (N+m) \Delta\xi + \frac{1}{2} (\theta_0 - \theta_i) \left[ 2 \left( \frac{j \Delta\tau}{\pi} \right)^{1/2} \left\{ e^{-\frac{(2N+m)^2}{4\sigma j}} - e^{-\frac{m^2}{4\sigma j}} \right\} + (N+m) \Delta\xi \left\{ \operatorname{erfc} \frac{m}{2\sqrt{\sigma j}} - \operatorname{erfc} \frac{2N+m}{2\sqrt{\sigma j}} \right\} \right]. \quad (19)$$

When the numerical solutions were compared with those given by equations (18) and (19) it was found that nowhere did the error exceed 0.1% with the use of either boundary equation and that, in general, the actual error was approximately halved when (12') was used. It will be seen from Fig. 2, where  $\tau_e$  is plotted against  $\Delta\tau$  for both boundary equations, that equation (12) led to faster ignition while both equations give a common value of  $\tau_e$  as  $\Delta\tau \rightarrow 0$ . Thus the use of equation (12') as the boundary condition saves much computation time while the dependence of  $\tau_e$  on  $\Delta\tau$  appears to be linear, making extrapolation to the exact solution simpler. The results quoted later are derived by use of equation (12') in preference to equation (12).

It is also apparent from Fig. 2 that the ignition time is sensitive to changes in  $\Delta\tau$ . However, as demonstrated in Fig. 3 where results are obtained for two values of  $\Delta\tau$ , the form of the curves remains the same. Since this is expected to be generally the case, and in this report we are more concerned with the behaviour of  $t_e$  rather than exact numerical results, only one value of  $\Delta\tau$  has been taken for any derivation of  $t_e$ . In any case, there is some considerable doubt as to the accuracy of the published figures used for obtaining the parameters  $a_1$ ,  $a_2$ ,  $a_3$  of equation (9). If accurate ignition times are required these may be obtained by determining  $t_e$  for two or three values of  $\Delta\tau$  and extrapolating to zero in the manner of Fig. 2.

It has been shown by O'Brien, Hyman, and Kaplan [7] that linear equations put into finite difference form in the way described in para. 3. are stable if  $\sigma < \frac{1}{2}$  and it is known that the truncation error is at a minimum if  $\sigma = \frac{1}{6}$ . For comparison two values of  $\sigma$  were investigated,  $\sigma = \frac{1}{2}$  and  $\sigma = \frac{1}{4}$  while  $\Delta\tau$  was kept constant. The resulting values of  $t_e$  differed by less than 0.5%, and since it is convenient when considering small radii and relatively large times to choose  $\sigma$  as large as possible, the value  $\sigma = \frac{1}{2}$  was used in all further calculations.

## 5. RESULTS

Owing to the large number of parameters to be considered, machine computation was confined to give only specimen results, representative of a wide field. For this purpose four actual hot spot materials were selected with thermal properties ranging from those similar to the explosive itself to those of copper. The remaining computation time was devoted to studying the general behaviour of the time to ignition  $t_e$  with varying radius of hot spot  $a$ , initial temperatures  $T_0$  and  $T_i$  of the hot spot and explosive respectively, and the thermal parameters

$$\mu (= \sqrt{\frac{K_2}{K_1}}), \quad \nu (= \frac{k_2}{k_1}).$$

The variation of  $t_e$  with either  $T_0$  or  $T_i$  is similar in appearance to that of  $t_e$  with the radius  $a$ . The gradient for small initial temperatures is, however, not so steep, and in the ranges considered the graphs suggest a relation of the form

$$\log t_e = \frac{A}{T'} + B, \quad (20)$$

where  $A$  and  $B$  are constants and  $T'$  is either  $T_0$  or  $T_i$ . This relation has also been found, though under different circumstances, by Henkin [8] and Cook [5].

Since the thermal parameters  $k_1$ ,  $k_2$ ,  $K_1$ ,  $K_2$  appear only in the ratios  $\mu$ ,  $\nu$  it is sufficient to investigate the dependence of  $t_e$  on these two variables. It is apparent from Fig. 4 that, for small values,  $t_e$  has an almost equal dependence on either  $\nu$  or  $\mu^{-1}$ , or, in terms of the hot spot properties, on either  $K_1^{-1}$  or  $k_1$ . For large values the dependence on  $\mu^{-1}$  dominates. Another fact that emerges from Fig. 4 is that if accurate ignition times are required for a hot spot with thermal properties similar to those of the explosive (i.e.  $\mu \sim 1$ ,  $\nu \sim 1$ ), more accurate knowledge of the explosive properties is needed than if the hot spot were made of a material such as steel.

The dominant variable is shown more clearly in Fig. 5 where  $t_e$  is plotted as a function of  $\mu$  under the condition  $\mu = \nu$ . For small values of  $\mu$ , even though the hot spot conductivity is extremely large,  $t_e$  approaches the adiabatic ignition time (i.e. the time taken for a large mass of explosive to ignite spontaneously). Here a relation of the form (20) exists where  $\mu$  is substituted for  $T'$ . At the other end of the scale it is apparent that, if  $k_1$  approximates to zero, it is still possible to obtain a small ignition time by supplying sufficient initial heat.

The four representative materials chosen for the hot spot were copper, steel, glass and a further material, substance A, having thermal properties identical to those of the explosive. The thermal properties assumed for the first three are listed in Table 1.

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Table 1

Material	$k_1$ (cal. $\text{cm}^{-1} \cdot \text{sec}^{-1} (\text{ }^\circ\text{C})^{-1}$ )	$K_1$ ( $\text{cm}^2 \cdot \text{sec}^{-1}$ )
Copper	0.85	0.81
Steel	0.085	0.081
Glass	0.0025	0.006

The conductivity and diffusivity of the explosive were taken to be  $7 \times 10^{-4} \text{ cal.cm}^{-1} \text{ sec}^{-1} (\text{ }^\circ\text{C})^{-1}$  and  $10^{-3} \text{ cm.}^2 \text{ sec}^{-1}$  respectively in all cases. Figs. 6 and 7 show typical curves for ignition times against initial hot spot temperatures, the radius of the hot spot being 0.038 cms., the initial explosive temperature  $50^\circ\text{C}$  and the explosive RDX. The curves again approximate to a relation of the form of equation (20).

Since  $\mu$  and  $\nu$  are the only thermal parameters to appear in the dimensionless formulation of the problem it follows that, if these may be assumed constant for different explosives, results applicable to these explosives may be derived by choosing  $a_1$ ,  $a_2$ ,  $a_3$  of equation (9) accordingly. The values of these parameters adopted in this report for three explosives which approximately satisfy this condition are (Rideal and Robertson [2]):

Table 2

	RDX	PETN	HMX
$10^4 a_1$	0.418	0.423	0.377
$a_2$	1700	4700	8983
$a_3$	2770	20800	53800

Fig. 8 shows the dependence of  $t_e$  on the initial hot spot temperature  $T_0$  for each of the three explosives. For the curves labelled I, II and III, appropriate data are listed in the following table.

Table 3

Graph	Explosive	Radius of Hot Spot (cms.)	Initial temp. of Explosive ( $^\circ\text{C}$ )	Time scale factor (s)
I	RDX	0.5	15	$10^2$
II	PETN	0.181	11.6	$10^3$
III	HMX	0.095	46.3	$10^3$

The graphs in which  $t_e$  is plotted as a function of  $T_0$ ,  $T_i$  or  $a$ , all have a similar appearance in that, after a certain point is reached, a small decrease in one of the independent variables yields a large increase in  $t_e$ . This leads one to ask if there is not a small interval over which it might be said that at one end ignition is due to the hot spot and at the other to a uniform rise in the explosive temperature. In this manner it would be possible to define a "critical" value of  $T_0$ ,  $T_i$  or  $a$  for a given solid hot spot similar to that defined by Rideal and Robertson [2] for an explosive hot spot. They define "critical temperature" to be that initial hot spot temperature at which heat evolution due to chemical

reaction within the hot spot in a time  $t$ , just balances the heat lost by conduction to the surrounding explosive in the same time. This condition is given by the relation

$$\frac{4}{3} \pi a^3 \rho t \frac{QZe^{-E/RT_0}}{RT_0} = \int_a^{\infty} 4\pi r^2 T_2 \rho c dr,$$

the temperature  $T_2$  on the right hand side being determined from the linearised heat conduction equation on the assumption that heat evolution in the cold explosive is negligible during the time  $t$ . Thus for a given value of  $t$ , a "critical" value of  $T_0$  may be derived. In the present case however such a definition is impracticable and a graphical investigation appears to be necessary. So far as the computations have been conducted the curves do not differ significantly from equation (20). With the present formulation of the problem, an investigation of the higher parts of the curves would involve an enormous amount of computation and it has not, at present, been attempted.

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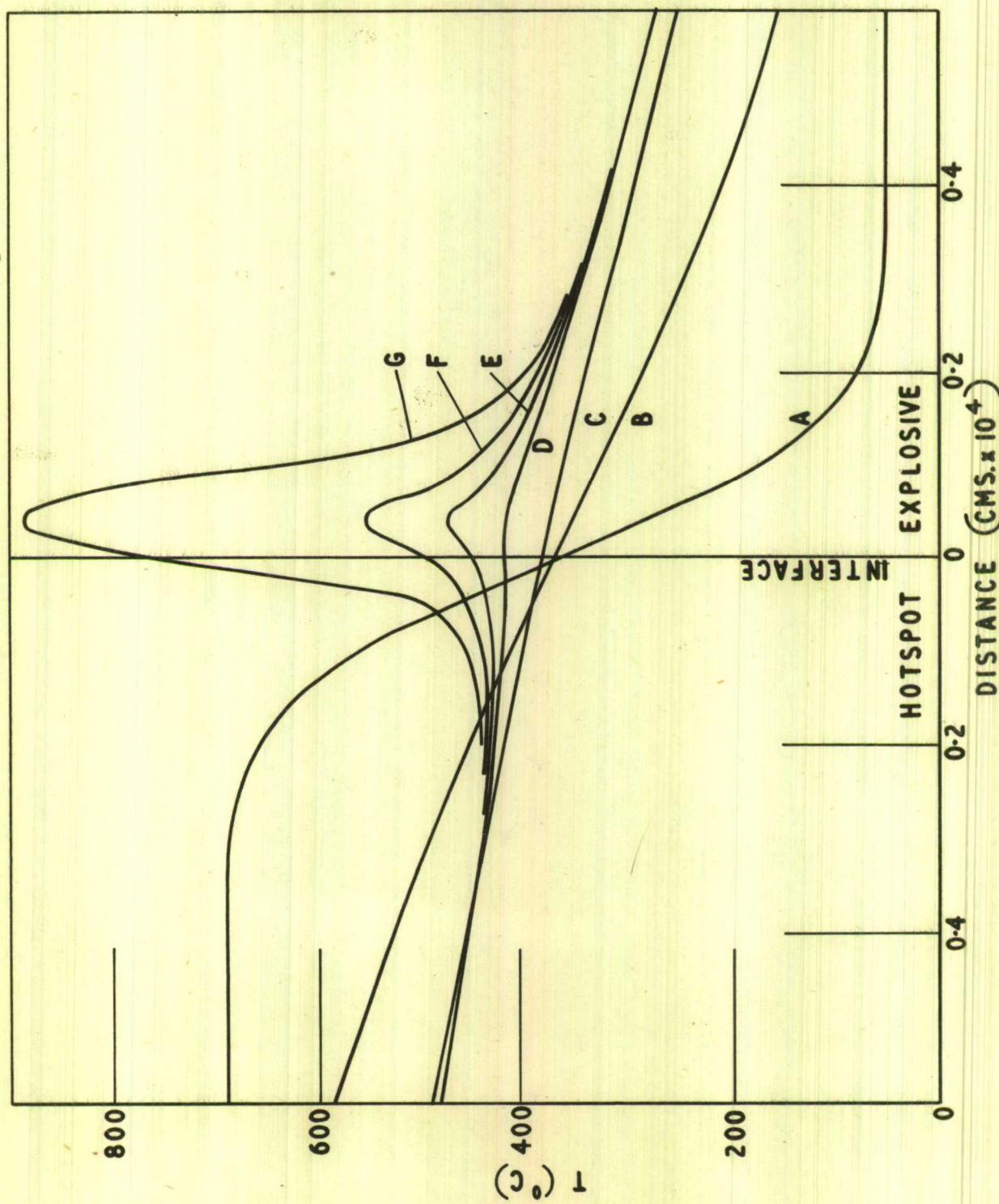
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FIG. I

A : INITIAL LINE  
B :  $0.18 \times 10^{-3}$  SECS.  
C : 0.72 " "  
D : 0.899 " "  
E : 0.910 " "  
F : 0.913 " "  
G : 0.917 "



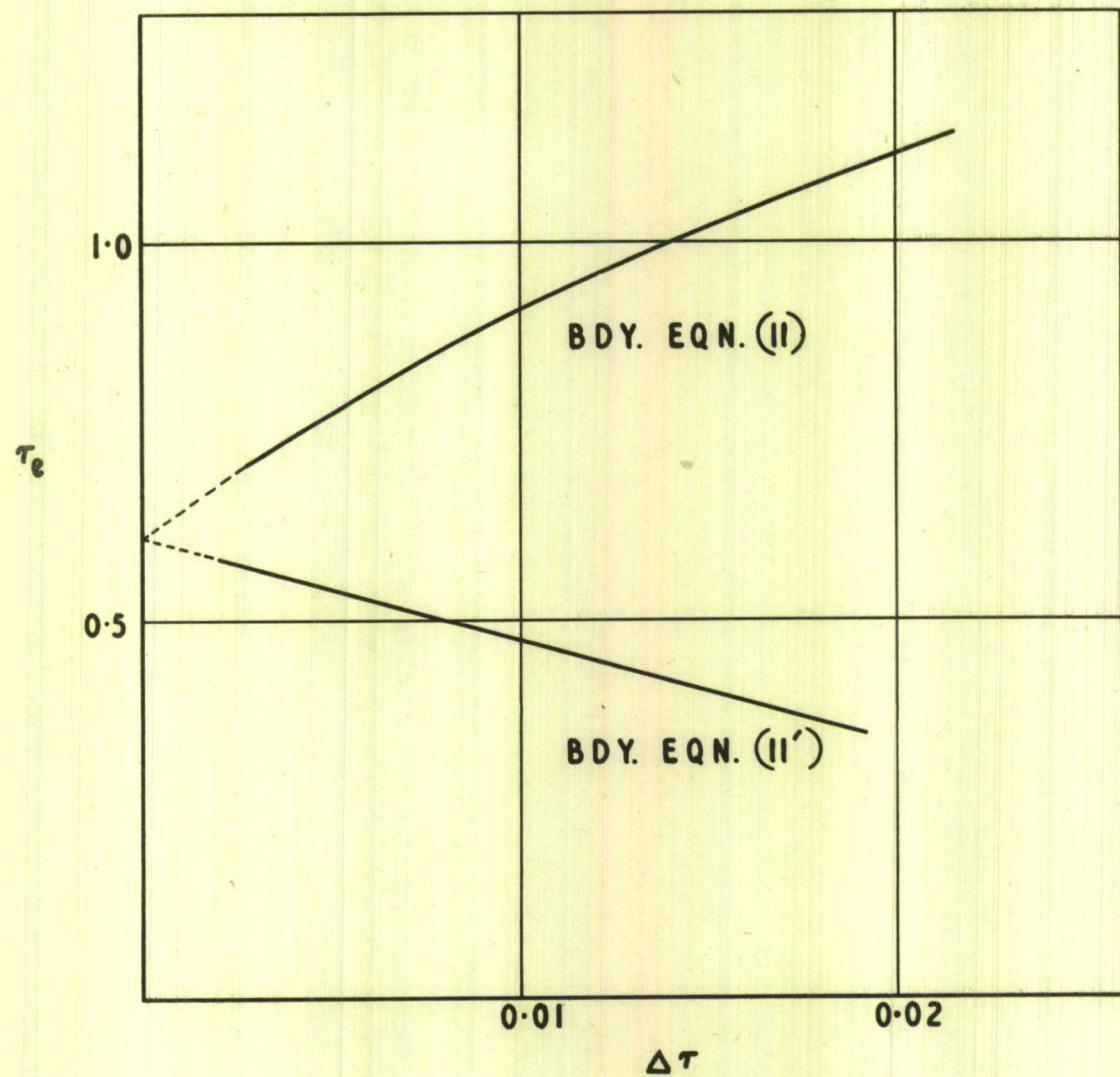


FIG. 2 IGNITION TIME AS A FUNCTION OF INTERVAL

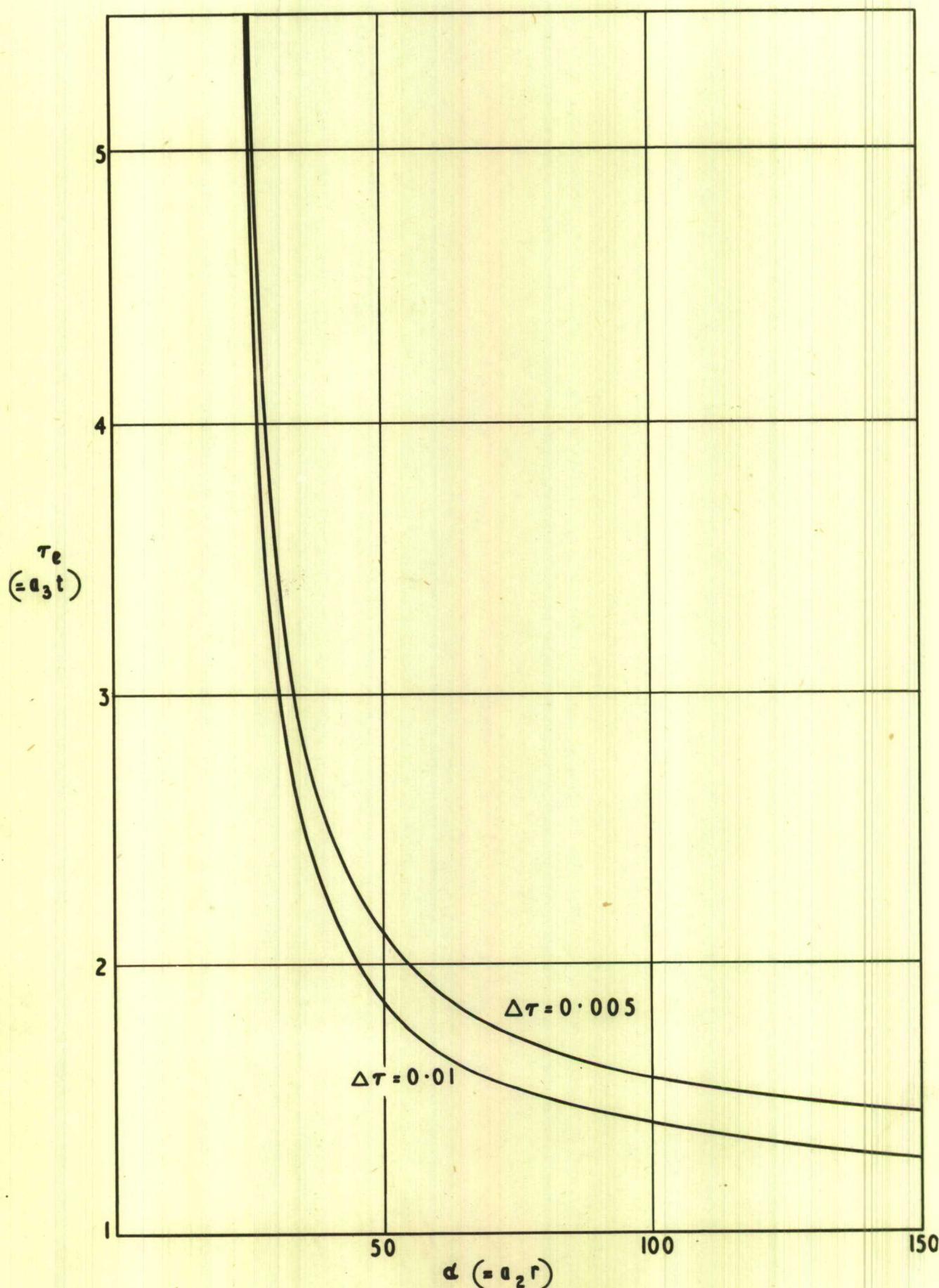


FIG. 3 IGNITION TIMES FOR VARIOUS RADII

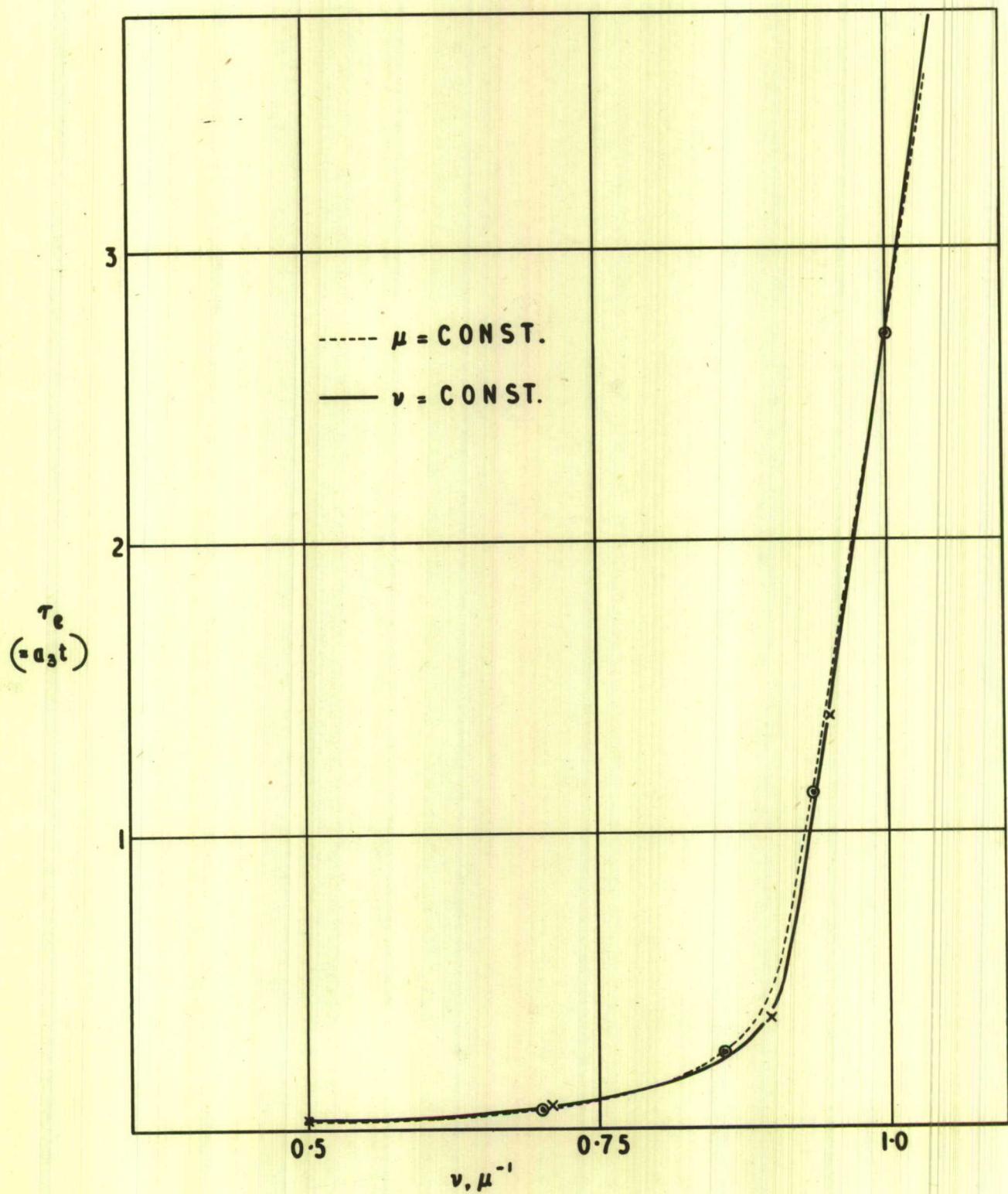


FIG. 4 IGNITION TIMES FOR VARYING RATIOS OF THE CONDUCTIVITIES,  $\nu$ , AND DIFFUSIVITIES,  $\mu^2$ , OF THE EXPLOSIVE AND HOT SPOT

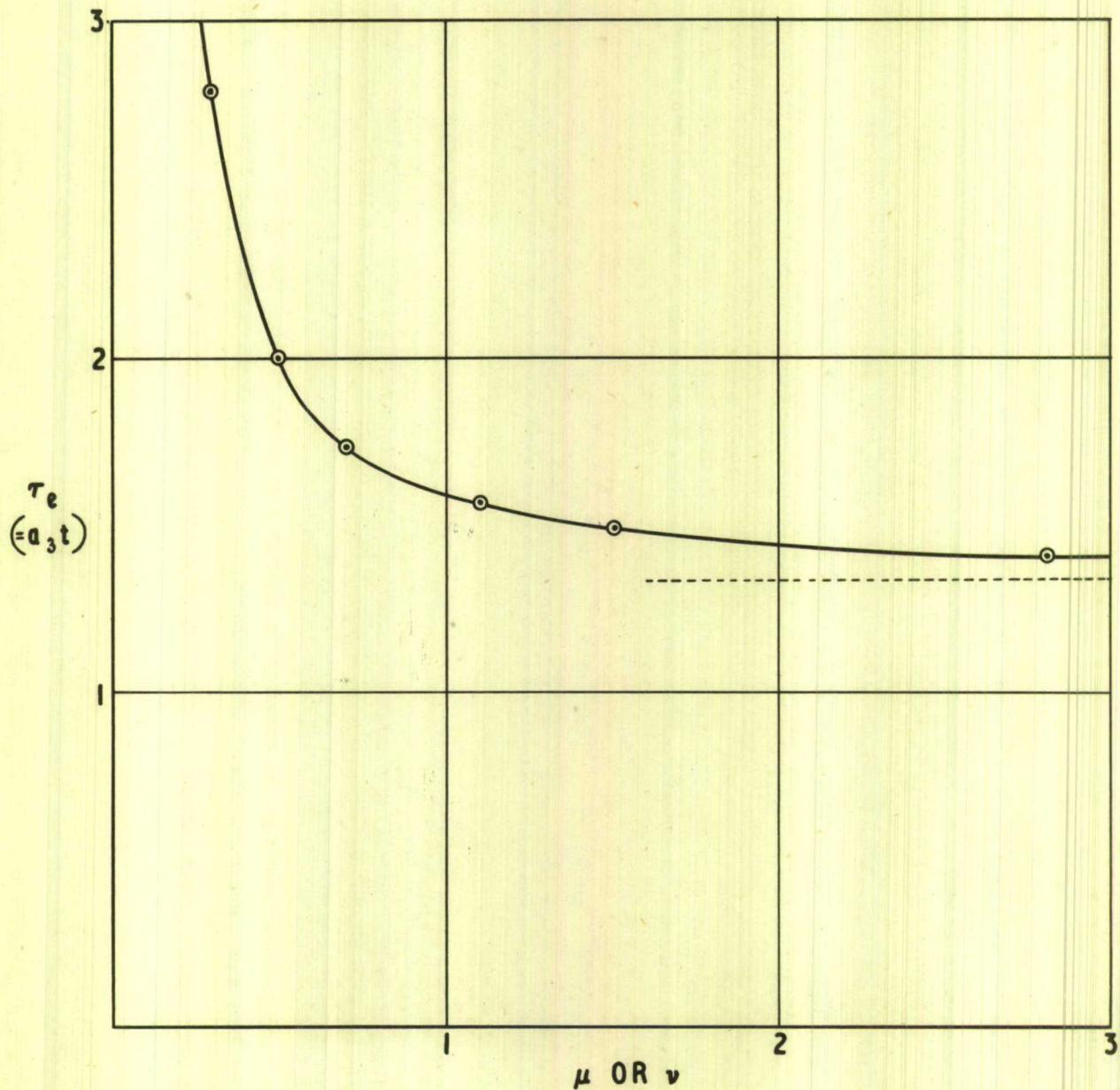


FIG.5 IGNITION TIMES WITH THE RATIO OF THE CONDUCTIVITIES OF THE HOT SPOT AND EXPLOSIVE EQUAL TO THAT OF THE DIFFUSIVITIES (i.e.  $\mu = \nu$ )

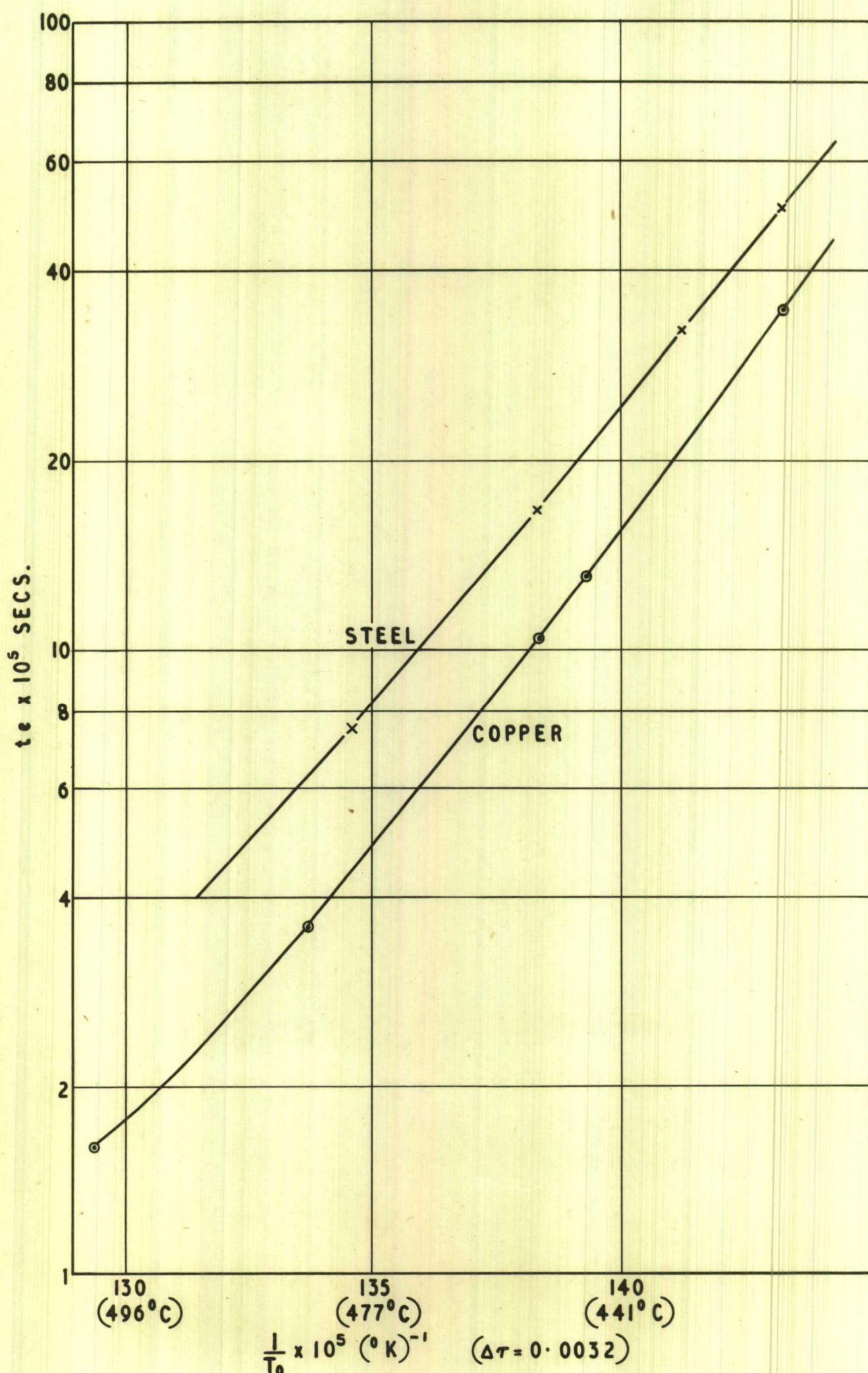


FIG. 6 IGNITION TIMES OF RDX FOR TWO TYPES OF HOT SPOT

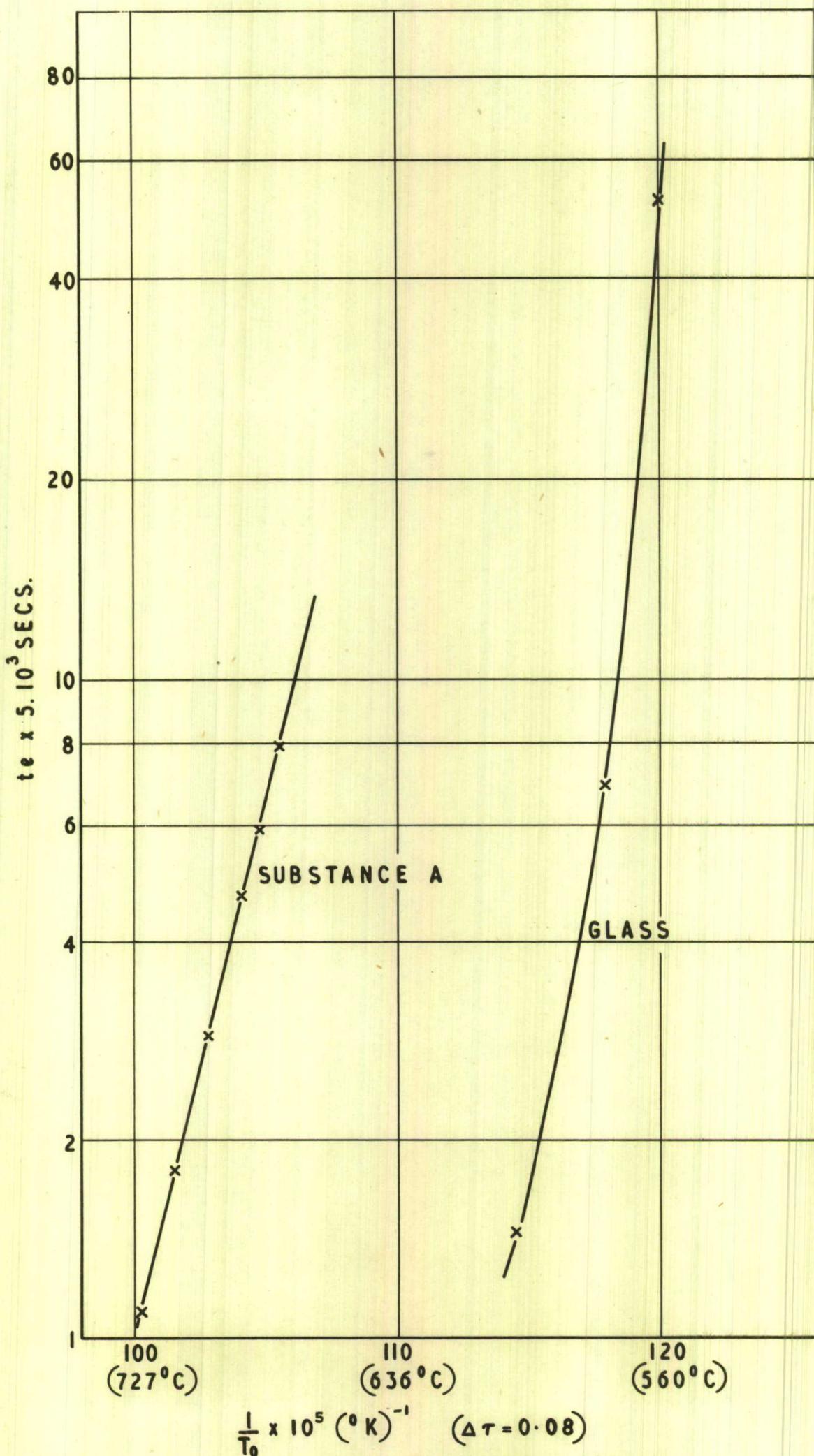


FIG.7 IGNITION TIMES OF RDX FOR TWO TYPES OF HOT SPOT

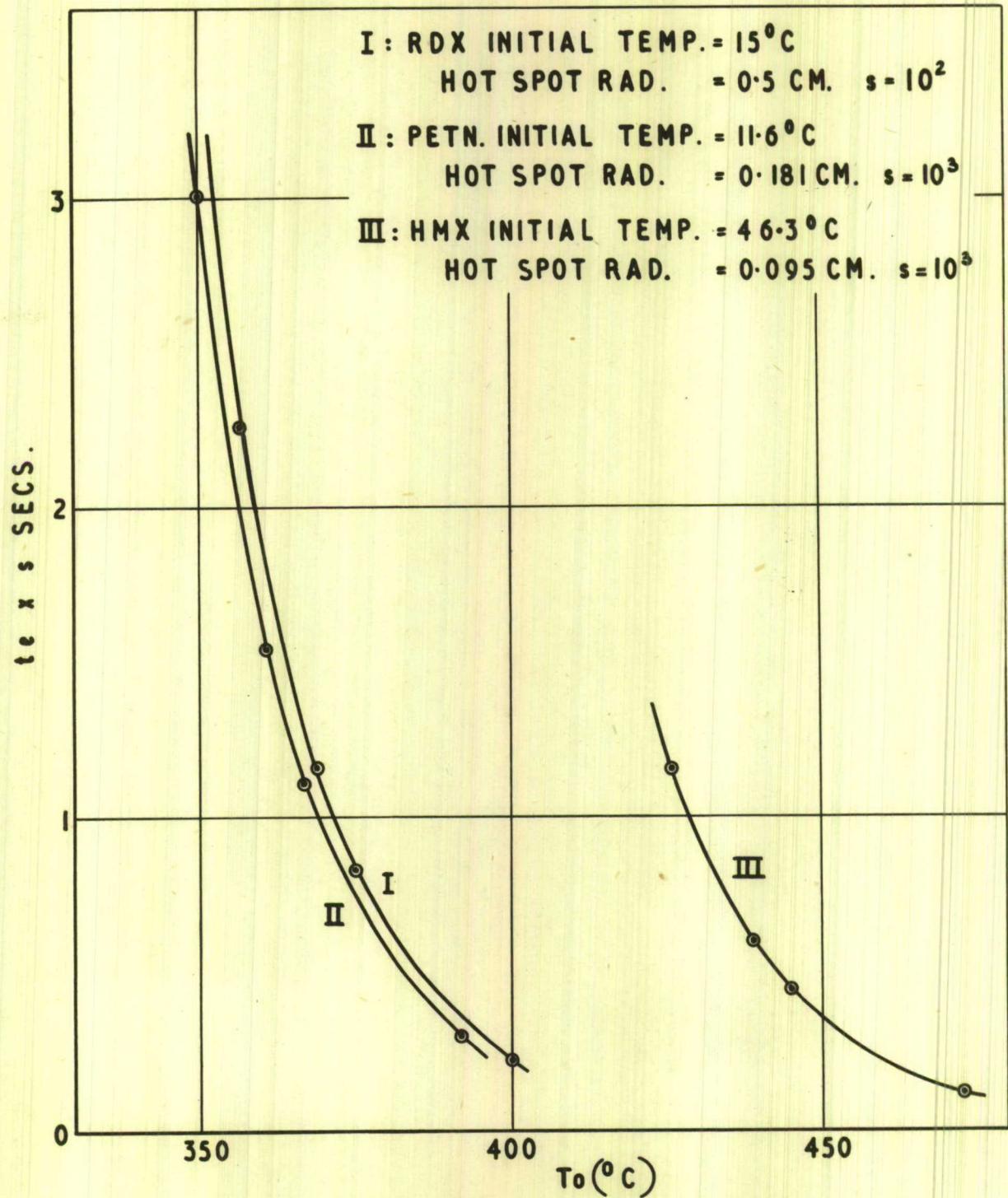


FIG.8 - IGNITION TIMES FOR THREE EXPLOSIVES WITH A STEEL HOT SPOT

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Sept. 1958

A small hot sphere of chemically inert material is embedded in a mass of cold explosive. The time to ignition of the explosive is determined by numerical methods, and the way in which this time varies with the properties of the hot spot and explosive is investigated. Four specific hot spot materials are considered: copper, steel, glass and a further material with thermal properties similar to those of the explosive. The first three of these were surrounded by RDX, but, if certain conditions are satisfied, the results are applicable to other explosives.

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